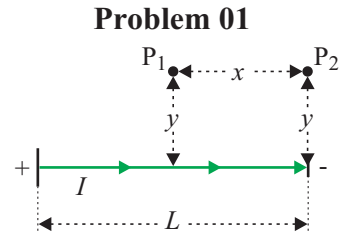# Chapter 16
Engineering Magnetism: Magnetic Field Calculations and Inductors
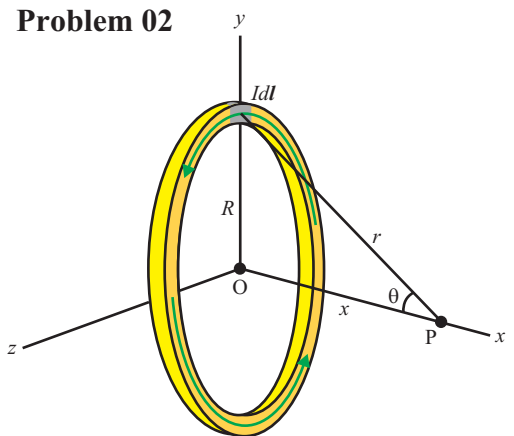## 16.1 The Biot-Savart Law (Magnetic Field Strength of Currents) Homework #140
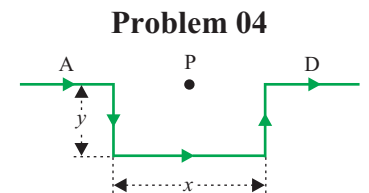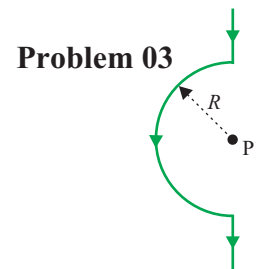
II

Problem 01

01. A short conducting wire is connected to a positive terminal on the left and a negative terminal on the right causing a current of I to flow to the right in the wire as shown in the diagram to the right. Point P_1 is on the perpendicular bisector of the wire, while point P_2 is directly above the right end of the wire. ($x = \frac{1}{2}L$)

    a.) What is magnitude (in terms of I, y, x, and any constants) and direction of the magnetic field at point P_1?

    b.) If $I = 12.0$ A, $y = 9.00$ cm and $L = 14.00$ cm, what is the strength of the magnetic field at P_1?

    c.) If $I = 12.0$ A, $y = 9.00$ cm and $L = 14.00$ cm, what is the strength of the magnetic field at P_2?

    d.) What is magnitude (in terms of I, y, x, and any constants) and direction of the magnetic field at point P_1 if $L \ggg y$?

    e.) If $I = 12.0$ A, $y = 9.00$ cm and $L = 14.00$ m, what is the strength of the magnetic field at P_1?

02. The diagram to the right shows a circular conducting loop with a current of I that is counterclockwise when viewed from the right as shown by the green arrows. The origin of the coordinate system, O, is at the center of the loop with the x-axis as the perpendicular bisector of the face of the loop. The distance between points O and P is x.

Problem 02

    a.) What is the magnitude (in terms of I, R, and any constants) and direction of the magnetic field, created by the current loop, at point O?

    b.) If $I = 7.00$ A and $R = 4.00$ cm, what is the strength of the magnetic field at O?

    c.) What is the magnitude (in terms of I, R, x, and any constants) and direction of the magnetic field at point P?

    d.) If $I = 7.00$ A, $R = 4.00$ cm, and $x = 7.50$ cm, what is the strength of the magnetic field at P?

03. A conducting wire is shaped as shown in the diagram to the right. The straight segments of the wire have a length of 6.00 cm each while the center portion is a semicircle with a radius of 8.00 cm. The current in the wire is in the direction shown and has a magnitude of 9.00 A. What is the magnitude and direction of the magnetic field created by this current-carrying wire at point P?

Problem 03

04. A conducting wire is shaped as shown in the diagram to the right. The current in the wire is in the direction shown and has a magnitude of 6.00 A. Wire segments A and D each have a length of 10.00 cm each. The two vertical segments (one of which is labeled y) each have a length of 8.00 cm each, while the segment labeled x has a length of 16.00 cm. What is the magnitude and direction of the magnetic field created by this current-carrying wire at point P?

Problem 04

ANSWERS: **01.** a.) $\dfrac{\mu_o}{2\pi}\dfrac{I}{y}\dfrac{x}{\sqrt{x^2+y^2}}$ (out of page)   b.) 1.64×10^{-5} T   c.) 1.12×10^{-5} T   d.) $\dfrac{\mu_o}{2\pi}\dfrac{I}{y}$ (out of page)

01. e.) 2.67×10^{-5} T   **02.** a.) $\dfrac{\mu_o}{2}\dfrac{I}{R}$ (out of page)   b.) 1.10×10^{-4} T   c.) $\dfrac{\mu_o}{2}\dfrac{IR^2}{\left(x^2+R^2\right)^{3/2}}$ d.) 1.15×10^{-5} T

03. 3.53×10^{-5} T (out of page)   **04.** 2.12×10^{-5} T (out of page)
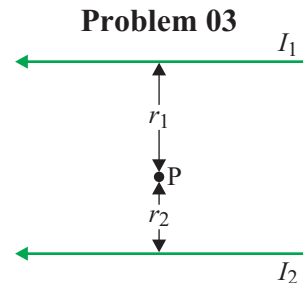
I

01. A current of 6.35 A is flowing through to the left in a long straight conducting wire as shown in the diagram to the right. If point P is located a distance of 3.85 cm from the wire, determine the magnitude and direction of the magnetic field at point P.

Problem 01

II

02. A solenoid, shown in the diagram to the right, has a length $L = 17.5$ cm, a diameter of $d = 2.30$ cm, and $N = 630$ turns (assume the windings of the solenoid are wrapped much tighter than depicted in the diagram such that the space between coils is very small). A current of $I = 4.75$ A is flowing through the solenoid. The rectangle MNOP is chosen as the closed path for Ampere's law. Segments P and N of the rectangle have a length of l, while segments M and O have a width w.
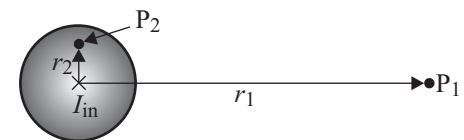
Problem 02

   a.) What is the number of turns per unit length, n?
   b.) What is the result of the integration of Ampere's law to segment M of the rectangular closed path?
   c.) What is the result of the integration of Ampere's law to segment N of the rectangular closed path?
   d.) What is the result of the integration of Ampere's law to segment O of the rectangular closed path?
   e.) What is the result of the integration of Ampere's law to segment P of the rectangular closed path?
   f.) What is the **TOTAL** current enclosed by rectangle MNOP?
   g.) What is the magnetic field strength, B, within the solenoid?

Problem 03

03. Two long parallel wires are shown in the diagram to the right. The current in the top wire is $I_1 = 4.00$ A to the left, while the current in the bottom wire is $I_2 = 5.50$ A to the left. Point P is positioned such that $r_1 = 7.00$ cm from the top wire, while $r_2 = 3.00$ cm from the bottom wire. What is the magnitude and direction of the net magnetic field at point P?

Problems 04 and 05 below refer to the diagram to the right and below which shows a wire with a diameter of 1.500 cm and a current of $I_{in} = 1.75$ A directed into the page. Point P_1 is a distance of $r_1 = 4.500$ cm from the center of the wire, while point P_2 is a distance of $r_2 = 0.500$ cm from the center.

04. The wire has a uniform current density, J.
   a.) What is the magnitude of the current density, J, within the wire?
   b.) What is the magnitude and direction of the magnetic field at point P_1?
   c.) What is the magnitude and direction of the magnetic field at point P_2?

Problems 04 and 05

III

05. The current density within the wire varies with distance from the center, r, according to the relationship $J = (9400 + 100{,}595r)$ A/m^2.
   a.) What is the total current $I_{enclosed}$ within an appropriate Ampere's law closed path to determine the B_{net} at point P_2?
   b.) What is the strength of the magnetic field, B_{net}, at point P_2?

ANSWERS: **01.** 3.30×10^{-5} T   **02.** a.) 3600 turns/m   b.) 0   c.) 0   d.) 0   e.) $B \cdot l$ f.) $n \cdot I \cdot l$ g.) 0.0215 T
03. 2.52×10^{-5} T into the page   **04.** a.) 9900 A/m^2 b.) 7.78×10^{-6} T toward bottom of page
04. c.) 3.11×10^{-5} T to the right   **05.** a.) 0.765 A   b.) 3.06×10^{-5} T
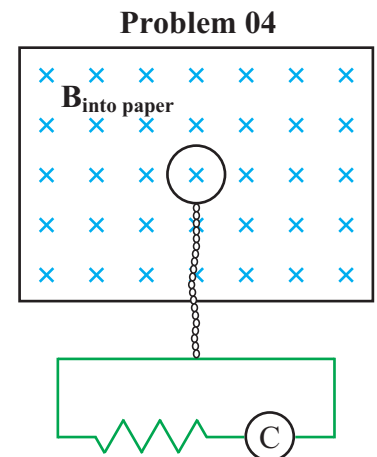
I

01. A 60-turn circular coil has a radius of 4.0 cm and a resistance of 15.0 Ω. What is the rate at which a magnetic field perpendicular to the face must change to produce a current of 2.50 A?

02. The flux through a loop is given by $\phi_m = (0.100 \ \text{Tm}^2/\text{s}) \cdot (3t^2 - 8t)$.
   a.) What is ϕ_m at $t = 0.00$ s?
   b.) What is ϕ_m at $t = 2.00$ s?
   c.) What is ϕ_m at $t = 4.00$ s?
   d.) What is ϕ_m at $t = 6.00$ s?
   e.) Write the expression for the emf, ε, as a function of time.
   f.) What is ε at $t = 0.00$ s?
   g.) What is ε at $t = 2.00$ s?
   h.) What is ε at $t = 4.00$ s?
   i.) What is ε at $t = 6.00$ s?

II

Problem 03

03. A magnetic field **B** is perpendicular to the plane of the page and uniform in a circular region of radius R as shown in the diagram to the right with the black circular border. Outside the circular region, **B** = 0. The magnitude of **B** is increasing at the rate of dB/dt. The red concentirc circle of radius r represents a region inside the black border that serves as a visual aid in calculations below.
   a.) Write an equation that describes the magnitude of the electric field, E, as a function of distance, r, from the center, where $r < R$.
   b.) Describe the direction of the electric field at all points a given distance, r, from the center, where $r < R$.
   c.) If the magnetic field strength is increasing at a rate of 0.250 T/s and $R = 12.00$ cm, what is E at 9.00 cm?
   d.) Write an equation that describes the magnitude of the electric field, E, as a function of distance, r, from the center, where $r > R$.
   e.) If the magnetic field strength is increasing at a rate of 0.250 T/s and $R = 12.00$ cm, what is E at 25.00 cm?

Problem 04

04. A flip coil is a small coil that is rotated in a magnetic field and is used to measure the strength of the magnetic field. The field strength is determined by connecting the flip coil to a device known as a current integrator, ©, which measures the amount of charge that passes through the coil as a result of the induced emf and current produced by the changing magnetic flux through the coil as it rotates. One such circular flip coil of N turns, area A, and resistance, R, is shown in the diagram to the right. The coil has its face in the plane of the page and perpendicular to a uniform magnetic field **B** which is into the plane of the page. Find the charge passing through the coil if the coil is rotated 180° about its diameter.

ANSWERS: **01.** 124 T/s   **02.** a.) 0 Tm^2 b.) -0.400 Tm^2 c.) 1.60 Tm^2 d.) 6.00 Tm^2
02. e.) $\varepsilon = 0.100 \cdot (6t - 8)$ f.) -0.800 V   g.) 0.400 V   h.) 1.60 V   i.) 2.80 V

03. a.) $E = \dfrac{r}{2}\dfrac{dB}{dt}$ b.) Tangent to a counterclockwise circle   c.) 11.3 mV   d.) $E = \dfrac{R^2}{2r}\dfrac{dB}{dt}$ e.) 7.20 mV
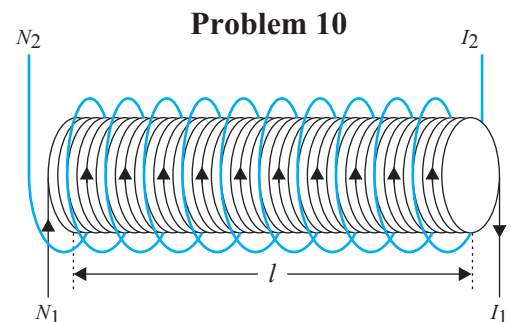
04. $Q = \dfrac{2NBA}{R}$

I

01. What is the induced emf in a 385-mH coil if the current varies from 15.0 mA to 40.0 mA in 425 ms?

02. An emf of 9.25 V is produced in a coil when its current changes from -42.0 mA to +57.0 mA in 37.4 ms. What is its inductance, L?

03. An air-filled coil that is 32.0 cm long is 2.48 cm in diameter and contains 7500 loops. What is its inductance, L?

04. A coil that is 12.5 cm long and has a diameter of 3.70 cm has an inductance of 1.95 mH when filled with air.
    a.) How many turns does are in this inductor?
    b.) How much energy is stored in this inductor when the current is 7.25 A?
    c.) If an iron core with a $\mu = 3 \times 10^3 \mu_o$ is placed inside the coil, what would be the inductance?
    d.) How much energy is stored in the inductor in part c.) above when the current is 7.25 A?

05. How much energy is stored in a 70.0 mH inductor at an instant when the current is 4.50 A?

06. A solenoid is 18.5 cm long and has a diameter of 2.30 cm. The magnetic field inside this solenoid is 0.775 T. Approximately how much energy is stored in this field?

07. A current rises sharply from 0 to I in 3.75 ms inducing a 60-V emf in a 0.225 H inductor. What is the value of I?

II

08. The earth's magnetic field strength near its surface varies around the earth but averages about 0.50 G ($1 \text{ G} = 10^{-4}$ T).
    a.) Estimate the average energy density of this field near the earth's surface.
    b.) Estimate the total energy stored in this field in the first 5.0 km above the earth's surface.

09. In a laboratory, an electric field of 3.0×10^3 V/m and a magnetic field of 1.5 T are produced.
    a.) What is the energy density for the electric field?
    b.) What is the energy density for the magnetic field?
    c.) What electric field strength would be needed to produce the same energy density as the 1.5 T magnetic field?

10. Two concentric solenoids are shown in the diagram to the right. Solenoid 1 has N_1 turns on an iron core ($\mu = 6000\mu_o$), a radius of r_1, and an initial current of I_1 flowing through it in a counterclockwise direction when viewed from the right as shown. The current drops to 0 in time t. Solenoid 2, the larger solenoid shown with blue wire in the diagram, has N_2 turns and a radius of r_2. Both solenoids have a length of l.
    a.) What is the mutual inductance?
    b.) What is the mutual inductance if $N_1 = 600$ turns, $r_1 = 1.50$ cm, $I_1 = 3.00$ A, $t = 15.0$ ms, $N_2 = 150$ turns, $r_2 = 2.00$ cm, and $l = 32.0$ cm?
    c.) For the solenoids described in part b.) above, what is the induced emf in Solenoid 2? (What is its direction?)

Problem 10

N_2 I_2 N_1 l I_1

ANSWERS: **01.** 0.0226 V   **02.** 3.49 H   **03.** 0.107 H   **04.** a.) 425 turns   b.) 0.0512 J   c.) 5.85 H   d.) 154 J
05. 0.709 J   **06.** 18.4 J   **07.** 1.00 A   **08.** a.) 9.95×10^{-4} J/m^3 b.) 2.54×10^{15} J
09. a.) 3.98×10^{-5} J/m^3 b.) 8.95×10^5 J/m^3 c.) 4.50×10^8 V/m $[Bc = (1.5 \text{ T})c$, where c is the speed of light $]$
10. a.) $M_{12} = \mu\left(\dfrac{N_1 N_2}{l}\right)\pi r_1^2 = 6000\mu_o\left(\dfrac{N_1 N_2}{l}\right)\pi r_1^2$ b.) 1.50 H   c.) 300 V (CCW when viewed from right)
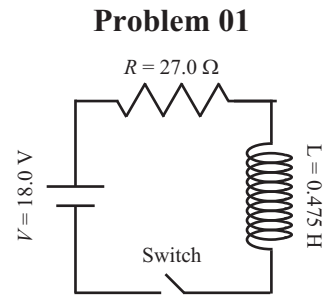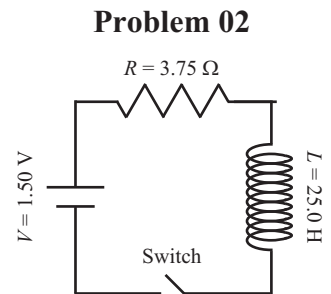
I

01. A circuit is constructed as shown in the diagram to the right. The voltage of the battery is $V = 18.0$ V, the resistor has a resistance of $R = 27.0$ Ω, and the inductor has a inductance of $L = 0.475$ H. The switch has been open for a long time, but closes as a clock starts at $t = 0$ s.

Problem 01

$R = 27.0\ \Omega$

$V = 18.0$ V

Switch

$L = 0.475$ H

a.) What is the initial current, I_0, of the circuit at $t = 0$ s? Explain!!!
b.) What is the time constant for this circuit?
c.) What will the current be a long time after the switch is closed?
d.) Write an equation that predicts the current as a function of time for this circuit.
e.) What will the current be at $t = 15.0$ ms?
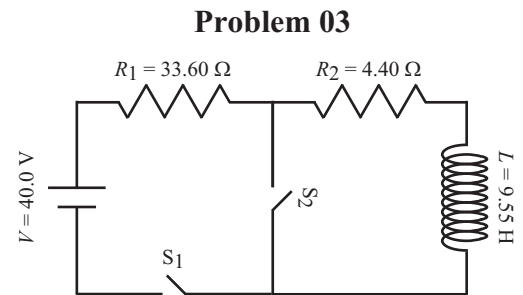f.) What will the current be at $t = 15.0$ s?   g.) Sketch a graph of the current as a function of time.

02. A circuit is constructed as shown in the diagram to the right. The voltage of the battery is $V = 1.50$ V, the resistor has a resistance of $R = 3.75$ Ω, and the inductor has a inductance of $L = 25.0$ mH. The switch has been closed for a long time so that a steady current was flowing in the circuit, but the switch opens as a clock starts at $t = 0$ s.

Problem 02

$R = 3.75\ \Omega$

$V = 1.50$ V

Switch

$L = 25.0$ H

a.) What is the initial current, I_0, of the circuit at $t = 0$ s? Explain!!!
b.) What is the time constant for this circuit?
c.) What will the current be a long time after the switch is opened?
d.) Write an equation that predicts the current as a function of time for this circuit.
e.) What will the current be at $t = 15.0$ ms?
f.) What will the current be at $t = 15.0$ s?   g.) Sketch a graph of the current as a function of time.

03. A circuit is constructed as shown in the diagram to the right. The voltage of the battery is $V = 40.0$ V, the two resistors have a resistances of $R_1 = 33.60$ Ω and $R_2 = 4.40$ Ω, and the inductor has a inductance of $L = 9.55$ H. Both switches have been open for a long time, but switch S_1 closes as a clock starts at $t = 0$ s, while switch S_2 remains open.

Problem 03

$R_1 = 33.60\ \Omega$ $R_2 = 4.40\ \Omega$

$V = 40.0$ V

S_2

S_1

$L = 9.55$ H

a.) What is the initial current, I_0, of the circuit at $t = 0$ s? Explain!!!
b.) What is the time constant for this circuit?
c.) What will the current be a long time after the switch is closed?
d.) Write an equation that predicts the current as a function of time for this circuit.
e.) What will the current be at $t = 15.0$ ms?   f.) Sketch a graph of the current as a function of time.
After a long time, S_1 opens as S_2 simultaneously closes, and a new clock starts ($t = 0$ s) at this instant.
g.) What is the initial current, I_0, of the circuit at $t = 0$ s? Explain!!!
h.) What is the time constant for this circuit?
i.) Write an equation that predicts the current as a function of time for this circuit.
j.) At what time does the current to drop to 10% of I_0?
k.) At what time does the current to drop to 1% of I_0?
l.) At what time does the current to drop to 0.1% of I_0? m.) Sketch a graph of the current as a function of time.

ANSWERS: **01.** a.) 0 A   b.) 17.6 ms   c.) 0.667 A   d.) $I = (0.667) \cdot (1 - e^{-t/0.0176})$ e.) 0.383 A   f.) 0.667 A

02. a.) 0.400 A   b.) 6.67 ms   c.) 0 A   d.) $I = (0.400) \cdot e^{-t/0.00667}$ e.) 0.0422 A   f.) 0 A

03. a.) 0 A   b.) 251 ms   c.) 1.05 A   d.) $I = (1.05) \cdot (1 - e^{-t/0.251})$ e.) 0.0610 A

03. g.) 1.05 A   h.) 2.17 s   i.) $I = (1.05) \cdot e^{-t/2.17}$ j.) 5.00 s   k.) 10.0 s   l.) 15.0 s